Hadamard 160 in Cool Tones

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Hadamard 160 in Cool Tones is a mathematical cross-stitch and the result of a collaboration between artist Bridjet Lee and mathematician Curtis Bright. Lee picked up cross-stitching as a hobby during the COVID-19 pandemic and subsequent 2020 lockdown restrictions in Ontario. After being introduced to visual representations of Hadamard matrices presented by scientists at NASA's Jet Propulsion Laboratory—in particular a framed Hadamard matrix appearing in a 1961 NASA press release [6]—she noticed the similarity between Hadamard matrices and cross-stitch patterns. With an interest in exploring the intersections of art and math, the collaborators selected one of the Hadamard matrices discovered in Bright's PhD thesis [2] on which to base the work.

The completed cross-stitch representing a 160×160 Hadamard matrix.

A *Hadamard matrix* is a square grid filled with two kinds of symbols such that every pair of rows "agree" (contain the same symbol) for exactly half of their entries and "disagree" (contain different symbols) for the other half of their entries. For example, rows □□⊠⊠ and □⊠□⊠ agree in the first and last entries and disagree in the second and third entries. First studied by Sylvester in 1867 (and then later by Hadamard in 1893), Hadamard matrices have since found a myriad of applications such as developing codes for communicating through space [5].

The Hadamard matrix represented in this work is of a special form devised by Williamson [8] and constructed via four "Williamson matrices" (each represented by a separate colour). Some theoretical constructions of Williamson matrices are known [1, 4, 7] but the lack of a general construction has motivated computational searches. The matrix in this work was found using satisfiability solving coupled with computer algebra to specify mathematical properties of Williamson matrices [3]—such as the fact that the Williamson matrices' squared rowsums must sum to the order of the Hadamard matrix (when representing the matrix entries by ±1). For example, the Williamson matrices used in this work produce the decomposition $0^2 + 0^2 + 4^2 + 12^2 = 160$. All decompositions which could potentially arise were enumerated using the computer algebra system Maple.

References

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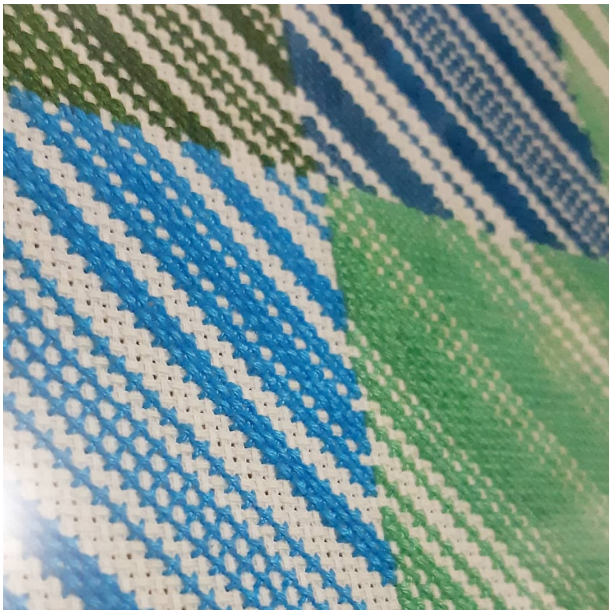

A closeup of the stitch work. The final piece was constructed with 14-count Aida cloth and embroidery floss in four colours. The work measures about 30cm × 30cm and comprises 12,800 stitches completed over six months.